Standard-Model Condensates and the Cosmological Constant

Stanley J. Brodsky[a,b] and Robert Shrock[b]
(a) Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309 and
(b) C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, NY 11794

We suggest a solution to the problem of some apparently excessive contributions to the cosmological constant from Standard-Model condensates.

PACS numbers: 95.36.+x, 98.80.-k, 98.80.Qc

One of the most challenging problems in physics is that of the cosmological constant Λ (recent reviews include [1]-[3]. This enters in the Einstein gravitational field equations as [4, 5]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = (8\pi G_N)T_{\mu\nu}, \tag{1}$$

where $R_{\mu\nu}$, R, $g_{\mu\nu}$, $T_{\mu\nu}$, and G_N are the Ricci curvature tensor, the scalar curvature, the metric tensor, the stress-energy tensor, and Newton's constant. One defines

$$\rho_\Lambda = \frac{\Lambda}{8\pi G_N} \tag{2}$$

and

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2} = \frac{\rho_\Lambda}{\rho_c} \ , \tag{3}$$

where

$$\rho_c = \frac{3H_0^2}{8\pi G_N} \ , \tag{4}$$

and $H_0 = (\dot{a}/a)_0$ is the Hubble constant in the present era, with $a(t)$ being the Friedmann-Robertson-Walker scale parameter [4, 6]. Long before the current period of precision cosmology, it was known that Ω_Λ could not be larger than O(1). In the context of quantum field theory, this was very difficult to understand, because estimates of the contributions to ρ_Λ from (i) vacuum condensates of quark and gluon fields in quantum chromodynamics (QCD) and the vacuum expectation value of the Higgs field hypothesized in the Standard Model (SM) to be responsible for electroweak symmetry breaking, and from (ii) zero-point energies of quantum fields appear to be too large by many orders of magnitude. Observations of supernovae showed the accelerated expansion of the universe and are consistent with the hypothesis that this is due to a cosmological constant, $\Omega_\Lambda \simeq 0.76$ [7, 8, 9].

Here we shall propose a solution to the problem of QCD condensate contributions to ρ_Λ. We also comment on other contributions of type (i) and (ii). Two important condensates in QCD are the quark condensates $\langle \bar{q}q \rangle \equiv \langle \sum_{a=1}^{N_c} \bar{q}_a q^a \rangle$, where q is a quark whose current-quark mass is small compared with the confinement scale $\Lambda_{QCD} \simeq 250$ MeV, and the gluon condensate, $\langle G_{\mu\nu}G^{\mu\nu} \rangle \equiv \langle \sum_{a=1}^{N_c^2-1} G^a_{\mu\nu} G^{a\,\mu\nu} \rangle$, where $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s c_{abc} A^b_\mu A^c_\nu$, a, b, c denote the color indices, g_s is the color $\mathrm{SU}(3)_c$ gauge coupling, $N_c = 3$, and c_{abc} are the structure constants for $\mathrm{SU}(3)_c$. These condensates form at times of order 10^{-5} sec. in the early universe as the temperature T decreases below the confinement-deconfinent temperature $T_{dec} \simeq 200$ MeV. For $T << T_{dec}$, in the conventional quantum field theory view, these condensates are considered to be constants throughout space. If this were true, then they would contribute $(\delta\rho_\Lambda)_{QCD} \sim \Lambda^4_{QCD}$, so that $(\delta\Omega_\Lambda)_{QCD} \simeq 10^{45}$. However, we have argued in Ref. [13] that, contrary to this conventional view, these condensates (and also higher-order ones such as $\langle (\bar{q}q)^2 \rangle$ and $\langle (\bar{q}q)G_{\mu\nu}G^{\mu\nu} \rangle$) have spatial support within hadrons, not extending throughout all of space. The reason for this is that the condensates arise because of quark and gluon interactions, and these particles are confined within hadrons [14]. We have argued that, consequently, these QCD condensates should really be considered as comprising part of the masses of hadrons. Hence, we conclude that their effect on gravity is already included in the baryon term Ω_b in Ω_m and, as such, they do not contribute to Ω_Λ.

Another excessive type-(i) contribution to ρ_Λ is conventionally viewed as arising from the vacuum expectation value of the Standard-Model Higgs field, $v_{EW} = 2^{-1/4}G_F^{-1/2} = 246$ GeV, giving $(\delta\rho_\Lambda)_{EW} \sim v^4_{EW}$ and hence $(\delta\Omega_\Lambda)_{EW} \sim 10^{56}$. Similar numbers are obtained from Higgs vacuum expectation values in supersymmetric extensions of the Standard Model (recalling that the supersymmetry breaking scale is expected to be the TeV scale). However, it is possible that electroweak symmetry breaking is dynamical; for example, it may result from the formation of a bilinear condensate of fermions F (called technifermions) subject to an asymptotically free, vectorial, confining gauge interaction, commonly called technicolor (TC), that gets strong on the TeV scale [15]. In such theories there is no fundamental Higgs field. Technicolor theories are challenged by, but may be able to survive, constraints from precision electroweak data. By our arguments in [13], in a technicolor theory, the technifermion and technigluon condensates would have spatial support in the technihadrons and techniglueballs and would contribute to the masses of these states. We stress that, just as was true for the QCD condensates, these technifermion and technigluon condensates would not contribute to ρ_Λ. Hence, if a technicolor-type mechanism should turn out to be responsible for electroweak

symmetry breaking, then there would not be any problem with a supposedly excessive contribution to ρ_Λ for a Higgs vacuum expectation value. Indeed, stable technihadrons in certain technicolor theories may be viable dark-matter candidates [18].

We next comment briefly on type-(ii) contributions. The formal expression for the energy density E/V due to zero-point energies of a quantum field corresponding to a particle of mass m is

$$E/V = \int \frac{d^3k}{(2\pi)^3} \frac{\omega(k)}{2} \ , \tag{5}$$

where the energy is $\omega(k) = \sqrt{\mathbf{k}^2 + m^2}$. However, first, this expression is unsatisfactory, because it is (quartically) divergent. In modern particle physics one would tend to regard this divergence as indicating that one is using an low-energy effective field theory, and one would impose an ultraviolet cutoff M_{UV} on the momentum integration, reflecting the upper range of validity of this low-energy theory. Since neither the left- nor right-hand side of eq. (5) is Lorentz-invariant, this cutoff procedure is more dubious than the analogous procedure for Feynman integrals of the form $\int d^4k \, I(k,p)$ in quantum field theory, where $I(k, p_1, ..., p_n)$ is a Lorentz-invariant integrand function depending on some set of 4-momenta $p_1, ..., p_n$. If, nevertheless, one proceeds to use such a cutoff, then, since a mass scale characterizing quantum gravity (QG) is $M_{Pl} = G_N^{-1/2} = 1.2 \times 10^{19}$ GeV, one would infer that $(\delta\rho_\Lambda)_{QG} \sim M_{Pl}^4/(16\pi^2)$, and hence $(\delta\Omega_\Lambda)_{QG} \sim 10^{120}$. With the various mass scales characterizing the electroweak symmetry breaking and particle masses in the Standard Model, one similarly would obtain $(\delta\Omega_\Lambda)_{SM} \sim 10^{56}$. Given the fact that eq. (5) is not Lorentz-invariant, one may well question the logic of considering it as a contribution to the Lorentz-invariant quantity ρ_Λ [19, 20]. Indeed, one could plausibly argue that, as an energy density, it should instead be part of T_{00} in the energy-momentum tensor $T_{\mu\nu}$. Phrased in a different way, if one argues that it should be associated with the $\Lambda g_{\mu\nu}$ term, then there must be a negative corresponding zero-point pressure satisfying $p = -\rho$, but the source for such a negative pressure is not evident in eq. (5).

The light-front (LF) quantization of the Standard Model provides another perspective. In this case, the Higgs field has the form [24] $\phi = \omega + \varphi$ where ω is a classical zero mode determined by minimizing the Yukawa potential $V(\phi)$ of the SM Lagrangian, and φ is the quantized field which creates the physical Higgs particle. The coupling of the leptons, quarks, and vector bosons to the zero mode ω give these particles their masses. The electroweak phenomenology of the LF-quantized Standard Model is in fact identical to the usual formulation [24]. In contrast to conventional instant-form Standard Model is trivial in the light-front formulation [22, 23], and there is no zero-point fluctuation in the light-front theory since ω is a classical quantity. Although this eliminates any would-be type-(ii) contributions of zero-point fluctuations to the cosmological constant, the contribution to the electroweak action from the Standard Model Yukawa potential $V(\omega)$ evaluated at its minimum would, as in the conventional analysis, yield an excessively large type-(i) contribution $(\delta\Omega_\Lambda)_{EW} \sim 10^{56}$. Thus the light-front formulation of the Standard Model based on a fundamental elementary Higgs field evidently does not solve the problem with type-(i) electroweak contributions to Ω_Λ. However, as we have noted above, theories with dynamical electroweak symmetry breaking, such as technicolor, are able to solve the problem with type-(i) contributions.

In summary, we have suggested a solution to what has hitherto commonly been regarded as an excessively large contribution to the cosmological constant by QCD condensates. We have argued that these condensates do not, in fact, contribute to Ω_Λ; instead, they have spatial support within hadrons and, as such, should really be considered as contributing to the masses of these hadrons and hence to Ω_b. We have also suggested a possible solution to what would be an excessive contribution to Ω_Λ from a hypothetical Higgs vacuum expectation value; the solution would be applicable if electroweak symmetry breaking occurs via a technicolor-type mechanism.

This research was partially supported by grants DE-AC02-76SF00515 (SJB) and NSF-PHY-06-53342 (RS). Preprint SLAC-PUB-13166, YITP-SB-08-09.

[1] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).

[2] S. Carroll, Liv. Rev. Rel. **4**, 1 (2001); T. Padmanabhan, Phys. Rept. **380**, 235 (2003); Gen. Rel. Grav. **40**, 529 (2008); E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006); V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **15**, 2105 (2006); S. Bludman, astro-ph/0702085; J. Frieman, M. S. Turner, and D. Huterer, Ann. Rev. Astron. Astrophys, in press (arXiv:0803.0982). An earlier review is S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).

[3] Dark Energy Task Force (A. Albrecht et al.), astro-ph/0609591.

[4] Recent texts on cosmology include P. Coles and F. Lucchin, *Cosmology: the Origin and Evolution of Cosmic Structure*, 2nd ed. (Wiley, New York, 2002); S. Dodelson, *Modern Cosmology* (Academic, Boston, 2003); S. Carroll, *Spacetime and Geometry: an Introduction to General Relativity* (Addison-Wesley, San Francisco, 2004). Earlier treatments include S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley, New York, 1972); C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravity* (Freeman, San Francisco, 1973); P. J. E. Peebles, *Principles of Physical*

Cosmology (Princeton Univ. Press, Princeton, 1993).

[5] We use units where $\hbar = c = 1$, and our flat-space metric is $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$.

[6] The field equations imply $(\dot{a}/a)^2 = H^2 = (8\pi G_N/3)\rho + \Lambda/3 - k/a^2$ and $\ddot{a}/a = -4\pi G_N(\rho+3p) + \Lambda/3$, where ρ = total mass/energy density, p = pressure, and k is the curvature parameter; equivalently, $1 = \Omega_m + \Omega_\gamma + \Omega_\Lambda + \Omega_k$, where $\Omega_m = 8\pi G_N \rho_m/H_0^2$, $\Omega_\gamma = 8\pi G_N \rho_\gamma/H_0^2$, and $\Omega_k = -k/(H_0^2 a^2)$.

[7] The supernovae data [8, 9], together with measurements of the cosmic microwave background radiation, galaxy clusters, and other inputs, e.g., primordial element abundances, have led to a consistent determination of the cosmological parameters [10]-[12]. These include $H_0 = 73 \pm 3$ km/s/Mpc, $\rho_c = 0.56 \times 10^{-5}$ GeV/cm^3 = $(2.6 \times 10^{-3}\ \mathrm{eV})^4$, total $\Omega_m \simeq 0.24$ with baryon term $\Omega_b \simeq 0.042$, so that the dark matter term is $\Omega_{dm} \simeq 0.20$. In the equation of state $p = w\rho$ for the "dark energy", w is consistent with being equal to -1, the value if the accelerated expansion is due to a cosmological constant. Other suggestions for the source of the accelerated expansion include modifications of general relativity and time-dependent $w(t)$, as reviewed in [1]-[3].

[8] A. G. Riess et al., A. J. **116**, 1009 (1998); J. L. Tonry et al., Ap. J. **594**, 1 (2003); A. G. Riess et al., Ap. J. **607**, 665 (2004); A. G. Riess et al., Ap. J. **659**, 98 (2007).

[9] S. Perlmutter et al., Ap. J. **517**, 565 (1999); R. A. Knop et al., Ap. J. **598**, 102 (2003); P. Astier et al., Astron. Astrophys. **447**, 31 (2006).

[10] D. N. Spergel et al. Ap. J. Suppl. **146**, 175 (2003); *ibid.* **170**, 377 (2007), http://www.gsfc.nasa.gov.

[11] M. Tegmark et al., Phys. Rev. D **69**, 103501 (2004); M. Tegmark et al., Phys. Rev. D **74**, 123507 (2006); W. J. Percival et al., Mon. Not. Roy. Astr. Soc. **381**, 1053 (2007).

[12] For minireviews, see http://pdg.lbl.gov.

[13] S. J. Brodsky and R. Shrock, arXiv:0803.2541[hep-th].

[14] As we noted [13], this is somewhat analogous to the fact that although formal statistical mechanics treatments of condensates and related order parameters in condensed matter physics take an idealized thermodynamic (infinite-volume) limit and obtain condensates that are constant throughout this infinite spatial volume, actual condensates and order parameters are experimentally measured to exist in, not outside of, the finite-volume samples wherein the particles responsible for them reside, such as a magnetization in a piece of iron.

[15] S. Weinberg, Phys. Rev. D **19**, 1277 (1979); L. Susskind, Phys. Rev. D **20**, 2619 (1979).

[16] S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979); E. Eichten and K. Lane, Phys. Lett. **B90**, 125 (1980).

[17] In accord with the Heisenberg uncertainty principle, a TC condensate also affects virtual processes occurring over times $t \lesssim 1/300\ \ \mathrm{GeV}^{-1}$. This is relevant for the generation of quark and lepton masses in TC/ETC theories.

[18] S. Nussinov, Phys. Lett. B **165**, 55 (1985); Phys. Lett. B **279**, 111 (1992); R. S. Chivukula and T. P. Walker, Nucl. Phys. B **329**, 445 (1990); J. A. Frieman and G. F. Giudice, Nucl. Phys. B **355**, 162 (1991); J. Bagnasco, M. Dine, S. D. Thomas, Phys. Lett. B **320**, 99 (1994); T. Appelquist, M. Piai, R. Shrock, Phys. Rev. D **69**, 015002 (2004); S. B. Gudnason, C. Kouvaris, and F. Sannino, Phys. Rev. D **74**, 095008 (2006); H. Davoudiasl, Phys. Lett. B **646**, 172 (2007). For recent reviews of dark matter, see, e.g., the Dark Matter 2008 Symposium, UCLA, at http://www.physics.ucla.edu/hep/dm08.

[19] This criticism of the conventional lore has also been made in [1] and [20].

[20] R. L. Jaffe, Phys. Rev. D **72**, 021301 (2005). See also [21].

[21] S. A. Fulling, K. A. Milton, P. Parashar, and A. Romeo, Phys. Rev. D **76**, 025004 (2007).

[22] S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky, Phys. Rept. **301**, 299 (1998).

[23] S. J. Brodsky, in *Proc. 58th Scottish Universities Summer School*, hep-ph/0412101.

[24] P. P. Srivastava and S. J. Brodsky, Phys. Rev. D **66**, 045019 (2002).